ECON 4113. HOMEWORK 4. 100 POINTS. DUE APRIL 6.

1. (100/3 points) Dixit, Exercise 11.1. Read Dixit chapters 10 and 11 skipping the stuff that is about continuous time models.

2. (100/3 points) Consider the modification of the Red-n-Black problem solved in class. We have an agent who lives for n periods. In period 1 he is endowed with x dollars. Each period he decides on a bet a, where $0 \leq a \leq x$. In the next period, depending on the realization of a coin flip he gets $x + a$ with probability $w > \frac{1}{2}$ or $x - a$ with probability $1 - w$. After this the betting repeats. The utility, the agent gets, is calculated in the end of the game. If in period n (in which no betting is happening, since it is the last period) the agent has x_n dollars then his utility is $u(x_n) = \ln x_n$. The agent (being in period 1) tries to solve the problem

$$\max E[u(X_n)]$$

where X_n is a random variable generated by the coin flips together with the decisions the agent makes on the way. Find the optimal betting strategy, using the same technique as in class.

3. (100/3 points) Consider the 3 period "Eat-or-Save" problem as discussed in class. In period 0 a consumer is given some capital $k_0 > 0$. He can consume it or save for the later periods. Consumer solves the following problem:

$$\max_{c_{0,1,2}; k_{1,2} \in \mathbb{R}} \quad \sum_{t=0}^{2} \beta^t \ln c_t$$
$$\text{s.t.} \quad k_{t+1} + c_t \leq k_t, \qquad t = 1, 2$$
$$c_2 \leq k_2$$
$$c_t, k_t \geq 0, \qquad t = 0, 1, 2$$

Formulate this problem as a sequence of one period maximization problems, using the technique shown in class. Find the optimal values V_0^*, V_1^*, V_2^* and optimal paths of capital and consumption $\{c_1^*, c_2^*, c_3^*, k_1^*, k_2^*, k_3^*\}$. If math becomes too messy, just express these variables in terms of some other, which are known.