

**ECON 4113. HOMEWORK 1 (100 POINTS). DUE TUESDAY FEBRUARY 13 IN CLASS.**

**For the definitions of terms in these problems use my lectures or Blume.**

1. (15 points) Consider a function  $(x, y, z) \mapsto (x^2 + 2y^2 + 3z^2)e^{-(x^2+y^2+z^2)}$  where  $x, y, z \in \mathbb{R}$  (here is another useful piece of notation: people use the symbol  $\mapsto$  when they don't want to give the function a name). Find all critical points (those where Jacobian derivative is 0).
2. (10 points) Find the determinant and the rank of the following matrix:

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 5 & 6 \\ 1 & 0 & 8 \end{pmatrix}$$

3. (15 points) Find out if the following matrix is positive definite, negative definite or indefinite:

$$\begin{pmatrix} 1 & 2 & 0 \\ 2 & 4 & 5 \\ 0 & 5 & 6 \end{pmatrix}$$

4. (25 points) Let  $F : \mathbb{R}^3 \rightarrow \mathbb{R}$  be given by  $F(x, y, z) = xye^{-z}$ . Find the Jacobian derivative  $J$  of  $F$ . If you think of  $J$  as a function, what are its domain and target spaces?
5. (35 points) Find the Jacobian derivative  $H$  of  $J$  from the previous exercise. How are  $H$  and  $F$  related? Find all critical points of  $F$  and determine which of them are local maxima, local minima and saddle points.